What is the difference between probability and odds?

* There is a 60% chance of rain this evening.

* The odds of the Detroit Red Wings winning the Stanley Cup are 16:1.
Recall: The probability of an event compares the favourable outcomes to the total possible outcomes. This represents a part:whole comparison.

Probability is normally expressed as a fraction in lowest terms, however, it can also be expressed as a percent, decimal or in words.

\[
\text{Probability} = \frac{\text{favorable outcomes}}{\text{total possible outcomes}}
\]

Ex: What is the probability of choosing a heart from a standard deck of 52 playing cards?

The odds in favour is the ratio of favourable outcomes to unfavourable outcomes.

\[
\text{Odds in Favour} = \frac{n(A)}{n(A')} = \frac{\text{fav}}{\text{unfav}}
\]

The odds against is the ratio of unfavourable outcomes to favourable outcomes

\[
\text{Odds Against} = \frac{n(A')}{n(A)} = \frac{\text{unfav}}{\text{fav}}
\]

*Hint: The odds against are the reciprocal of the odds in favour!

The odds are always expressed as a ratio in lowest terms (Part:Part).
Investigate the Math (p. 142 TB)

Suppose that, at the beginning of a regular CFL season, the Saskatchewan Roughriders are given a 25% chance of winning the Grey Cup.

a) What is the event in the situation described above?

b) Express the probability that this event will occur.

c) Describe the complement of this event.

d) Express the probability of the complement.

e) Write the odds in favour of the Roughriders winning the Grey Cup.

f) Write the odds against the Rough riders winning the Grey Cup.
Ex: Bailey holds all the hearts from a standard deck of 52 playing cards. He asks Morgan to choose a single card without looking.

Determine the odds in favour of Morgan choosing a face card.

Ex: Research shows that the probability of an expectant mother, selected at random, having twins is \(rac{1}{32}\).

a) What are the odds in favour of an expectant mother having twins?

b) What are the odds against an expectant mother having twins?
Ex: A computer randomly selects a university student’s name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57:43. Determine the probability that the randomly selected university student will be male.

Ex: A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records. Who should go first?

<table>
<thead>
<tr>
<th>Player</th>
<th>Attempts</th>
<th>Goals Scored</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellen</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>Brittany</td>
<td>17</td>
<td>10</td>
</tr>
</tbody>
</table>
Ex: A group of Grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5:2, and the odds against winning Zap are 7:3. Which game should Madison play?

3.3: Probability Using Counting Methods

Investigate the Math (p. 151):

As a volunteer activity, 10 students want to put on a talent show at a retirement home. To organize the show, 3 of these students will be chosen at random to form a committee. Victoria really wants to be on this committee, since her grandmother lives at the home. Each student’s name will be written on a slip of paper and placed in a hat. Then 3 names will be drawn.
a) Is Victoria’s name just as likely to be drawn as any other name? Explain.

b) Does the order in which the names are drawn matter? Explain.

c) In how many different ways can 3 names be drawn from a hat with 10 names?

d) In how many different ways can Victoria’s name be drawn with 2 other names?

e) What is the probability that Victoria’s name will be drawn?
Ex: A survey was conducted of 500 adults who wore Halloween costumes to a party. Each person was asked how he/she acquire the costume:

* 360 adults created their costumes
* 60 adults rented their costumes
* 60 adults bought their costumes
* 20 adults borrowed costumes

What is the probability that the first four people who were polled all created their costumes?

Consider the following:

* What counting strategy will you use? Is order important?

* What are the total number of possible outcomes? What are the number of favourable outcomes?

* What are the number of ways to choose 4 people from the group who created their own costumes?

* How would you use this information to determine probability?
Ex: If a 4-digit number is generated at random from the digits 2, 3, 5 and 7 (without repetition of the digit), what is the probability that it will be even?

Consider the following:

* What counting strategy will I use in this problem?

* What information do I need to determine the probability?

* How will I determine the total number of 4-digit numbers that can be created from these 4 digits?

** What condition must be met in order for the number to be even?

* How will I determine the total number of 4-digit even numbers?
Ex: Jamaal, Ethan and Alberto are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan and Alberto will place first, second, and third, in any order.

Ex: About 20 years after they graduated from high school, Blake, Mario and Simon met in a mall. Blake had two daughters with him, and he said he had three other children at home. Determine the probability that at least one of Blake's children is a boy.
Ex: Bob hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face down. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

Ex: There are 18 bikes in Marie’s spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Dough, Erica and Frankie each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all 6 friends will be in the same row, with Allison and Frankie at either end.
3.4: Mutually Exclusive Events

**Mutually Exclusive Events:**

- Two or more events that cannot occur at the same time.
- In Set Theory, this would describe the disjoint set.

Ex: The Sun rising and the Sun setting are mutually exclusive events.

\[ P(A \cup B) = P(A) + P(B) \]

** In a mutually exclusive set, \( n(A \cap B) = 0 \)

*** If two events are mutually exclusive, the sum of the probabilities and the complement will be 100%.

**Non-Mutually Exclusive Events:**

- Two or more events that can occur at the same time.
- In Set Theory, this would be described as intersecting sets.

Ex: A student who plays hockey as well as basketball is a non-mutually exclusive event.

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

or alternatively

\[ P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B) \]

** Use the Principle of Inclusion and Exclusion & Venn Diagrams**
Consider the following Venn Diagram where D represents students on the debate team and B represents the students on the basketball team.

How can we determine the probability of the event? Is this a mutually exclusive or non-mutually exclusive event?

Ex: Sam and Julie are playing a board game. If a player rolls a sum that is greater than 8 or a multiple of 5, the player gets a bonus of 100 points. Determine the probability that Julie will receive a bonus of 100 points on her next roll.

(Mutually Exclusive or Non-Mutually Exclusive?)
Ex: A school newspaper published the results of a recent survey:

Eating Habits:
Student Survey Results

- 62% skip breakfast
- 24% skip lunch
- 22% eat both breakfast and lunch

a) Are skipping breakfast and skipping lunch mutually exclusive events?

b) Determine the probability that a randomly selected student skips breakfast but not lunch.

c) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.
Ex: Reid’s mother buys a new washer and dryer set for $2500 with a 1-year warranty. She can buy a 3-year extended warranty for $450. Reid researches the repair statistics for this washer and dryer set and finds the data in the table below. Should Reid’s mother buy the extended warranty? Justify your decision.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>$P$(repair within extended warranty period)</th>
<th>Average Repair Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>washer</td>
<td>22%</td>
<td>$400</td>
</tr>
<tr>
<td>dryer</td>
<td>13%</td>
<td>$300</td>
</tr>
<tr>
<td>both</td>
<td>3%</td>
<td>$700</td>
</tr>
</tbody>
</table>

Ex: A car manufacturer keeps a database of all the cars that are available for sale at all the dealerships in Western Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both heated leather seats and a sunroof.
3.5/3.6: Independent & Dependent Events

To determine whether two events are independent, you must determine whether one event will affect the probable outcome of another.

If the events do not affect each other, then the events are independent.

\[ P(A \cap B) = P(A) \cdot P(B) \]

If one event does affect the other, then the events are dependent.

When you have dependent events, you will use conditional probability to calculate the probability of both events occurring.

\[ P(A \cap B) = P(A) \cdot P(B|A) \]

(Number of outcomes will be affected)

* In a situation with replacement, independent events are created
* In a situation without replacement, dependent events are created

Ex: What is the probability of rolling a 3 on a die and tossing heads on a coin? (Hint: A tree diagram may help!)

(Independent or Dependent)
Ex: Marie and Charlie are playing a die and coin game. Each turn consists of rolling a regular die and tossing a coin. Points are awarded for rolling a 6 on the die and/or tossing heads with the coin:

* 1 point for either outcome
* 3 points for both outcomes
* 0 points for neither outcome

Independent or Dependent?

Players alternate turns. The first player who gets 10 points wins.

a) Determine the probability that Marie will get 1, 3, or 0 points on her first turn.

b) Verify your results for part a)
Ex: All 1000 tickets for a charity raffle have been sold and placed in a drum. There will be two draws. The first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning ticket will be returned to the drum so that it might be drawn again. Max has bought five tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

**Independent or Dependent?**

Ex: According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

**Independent or Dependent?**
Ex: A hockey team has jerseys in three different colours. There are 4 green, 6 white and 5 orange jerseys in the hockey bag. Todd and Blake are given a jersey at random. Students were asked to write an expression representing the probability that both jerseys are the same colour. Which student correctly identified the probability and why?

**Independent or Dependent?**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tony</td>
<td>(\frac{2}{4})(\frac{2}{3})(\frac{2}{5})</td>
</tr>
<tr>
<td>Sam</td>
<td>(\frac{2}{4})+(\frac{2}{6})+(\frac{2}{5})</td>
</tr>
<tr>
<td>Lesley</td>
<td>(\frac{4}{15})(\frac{3}{14})+(\frac{6}{15})(\frac{5}{14})+(\frac{5}{15})(\frac{4}{14})</td>
</tr>
<tr>
<td>Dana</td>
<td>(\frac{4}{15})(\frac{4}{15})+(\frac{6}{15})(\frac{6}{15})+(\frac{5}{15})(\frac{5}{15})</td>
</tr>
</tbody>
</table>